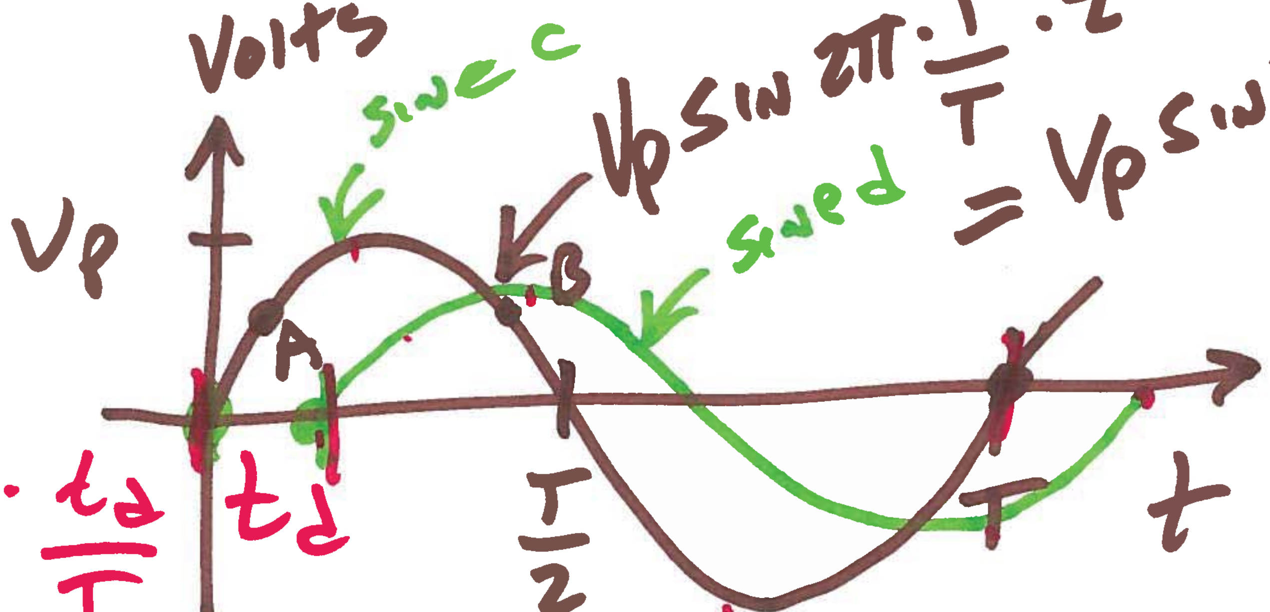
EE 220

Circuits 1

Lecture 21

$$f = \frac{1}{T}$$

$$V_p \sin 2\pi \cdot \frac{1}{T} \cdot t = V_p \sin 2\pi f \cdot t$$



$$\theta = 360^\circ \cdot \frac{t_d}{T}$$

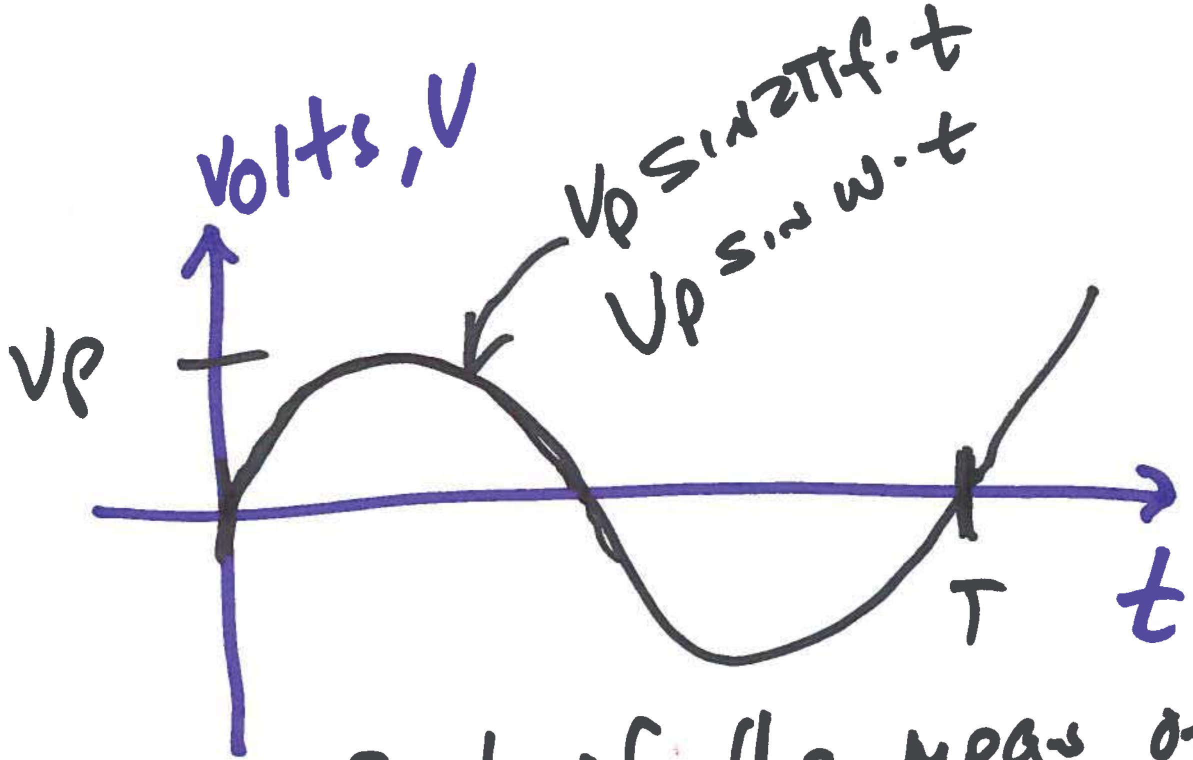
$$= 360 \cdot t_d \cdot f$$

sine c leads sine d

sine d lags sine c

1)

$$P = \frac{V^2}{R}$$

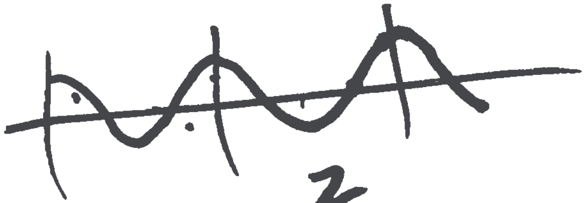


Root of the mean of the square

Root Mean Square

$$V_{rms} = \sqrt{\frac{V_p^2}{T}\int_0^T \sin^2(2\pi f\cdot t)\cdot dt}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$



$$V_{rms}^2 = \frac{V_p^2}{2T}\int_0^T \left(1-\cos 2\overset{2\pi f\cdot t}{x}\right)\cdot dt \qquad f=\frac{1}{T}$$

mean-squared value

$$= \frac{V_p^2}{2T}\left[\int_0^T dt - \int_0^T \cos \frac{4\pi}{T}\cdot t \cdot dt\right]$$

$$V_{p\cdot p} = \underline{\underline{339.4V}}$$

$$V_{rms}^2 = \frac{V_p^2}{2T}\left[T - 0\right]$$

$$V_p = \sqrt{2}\cdot 120\ rms$$

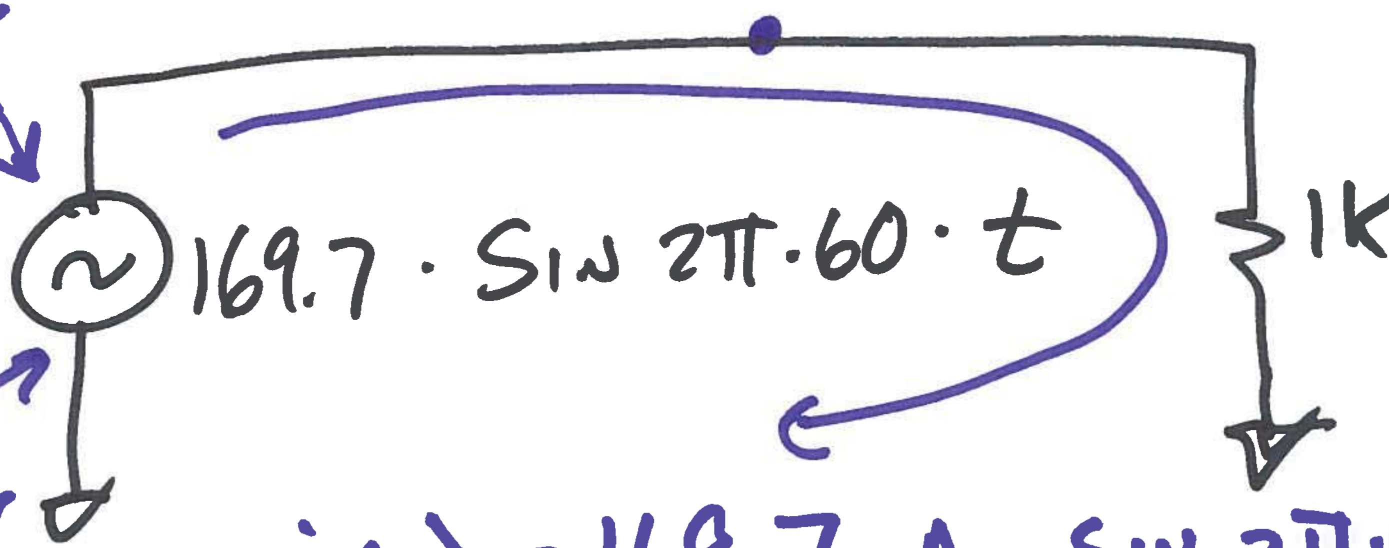
$$V_p = 169.7V$$

$$\boxed{V_{rms} = \frac{V_p}{\sqrt{2}}}$$

3)

$V_{PP} = 339.4\,V$

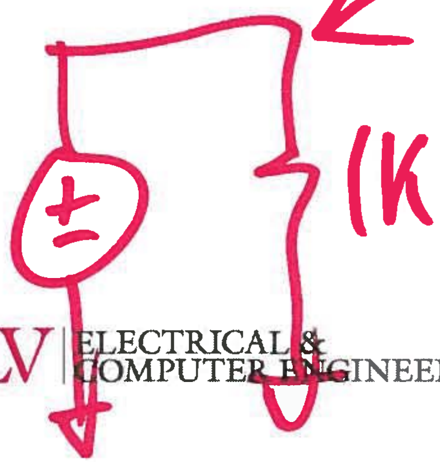
$v_r(t) = 169.7 \sin 2\pi 60 \cdot t$

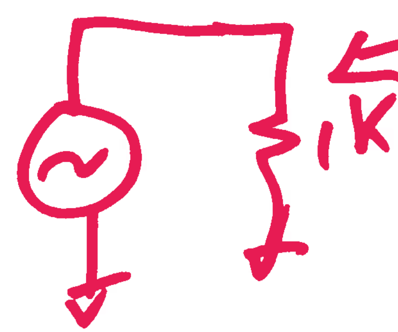


Rms = 120V

$i(t) = 169.7\,mA \cdot \sin 2\pi \cdot 60 \cdot t$

$$60\,Hz \rightarrow \underline{\underline{T = 16.67\,ms}}$$





$$P = \frac{V_{rms}^2}{1K} = \frac{120^2}{1K}\,W$$

18:30

Chapter

1

Signals, Filters, and Tools

Mixed-signal circuit design requires a fundamental knowledge of signals, signal processing, and circuit design. In this chapter we provide an overview of signals, filtering, and the mathematical tools. The chapter may be a review for the reader; however, we use it to ensure a good foundation to build on in the coming chapters and to provide a quick reference for the mathematical formulas we'll use throughout the book.

1.1 Sinusoidal Signals

Let's take a fundamental look at the sinewave. While there are many ways (equations and formulas) of representing a sinewave, we must remember it is an empirically determined function. Naturally occurring signals, shapes, or constants are determined or described through empirical measurements or observations. For example, π is determined by dividing the circumference of a circle by its diameter

$$\pi = \frac{\text{circumference}}{\text{diameter}} \tag{1.1}$$

The goal of this section is to provide intuitive discussions that will help create a deeper understanding of what's going on in a circuit or system.

1.1.1 The Pendulum Analogy

Consider the (ideal, that is, lossless) moving pendulum seen in Fig. 1.1a. In this figure the pendulum is moving back and forth between Points 1 and 3 repeatedly over time. As the pendulum leaves Point 1 it starts out slow, gaining maximum speed as it passes Point 2, and finally reaching Point 3. At Point 3 it stops and reverses direction. The time it takes to make this complete journey back to the starting point, Point 1 in this discussion, is the period, T. In Fig. 1.1b we plot the movement of the pendulum along the arched path. We record the position to define a function, $f(t)$, that indicates the pendulum's position at a specific time

$$Position = f(t) = f(t + nT), \text{ where } n \text{ is an integer} \tag{1.2}$$

This signal, we should all recognize, is a sinusoid or sinewave which repeats its position with a frequency, f_o, of $1/T$.

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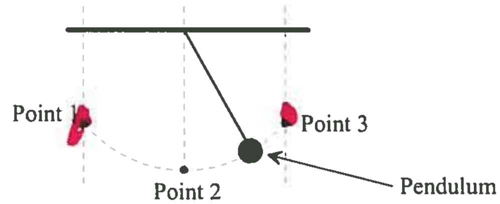
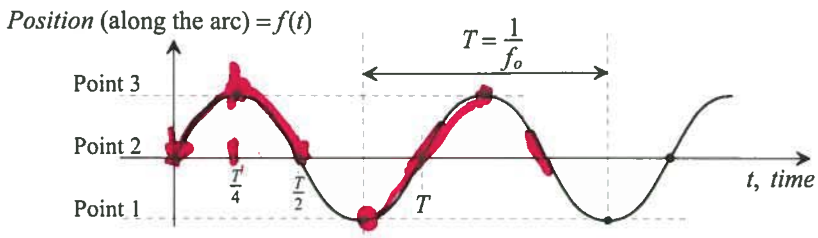


(a) Ideal (never stops or deviates from same path) swinging pendulum in motion. The time it takes to swing from Point 1 to Point 3 and back to Point 1 is the period T.



(b) Movement of pendulum along the arc in (a) over time.

Figure 1.1 Physical interpretation of a sinewave.

Next, consider the circle seen in Fig. 1.2. One complete rotation around this circle (360 degrees or 2π) is analogous to one complete movement (swing) of our pendulum. We started plotting the pendulum's position at Point 2 in Fig. 1.1b (Point 2, $t = 0$, in Fig. 1.2). After $T/4$ we reach Point 3 in Fig. 1.1b. This corresponds to a 90 degree, or $\pi/2$, movement in our circle. After another $T/4$ seconds we pass back through Point 2. In the circle we've moved 180 degrees. This continues with each swing of the pendulum corresponding to a complete revolution around the circle. Note that we do have some

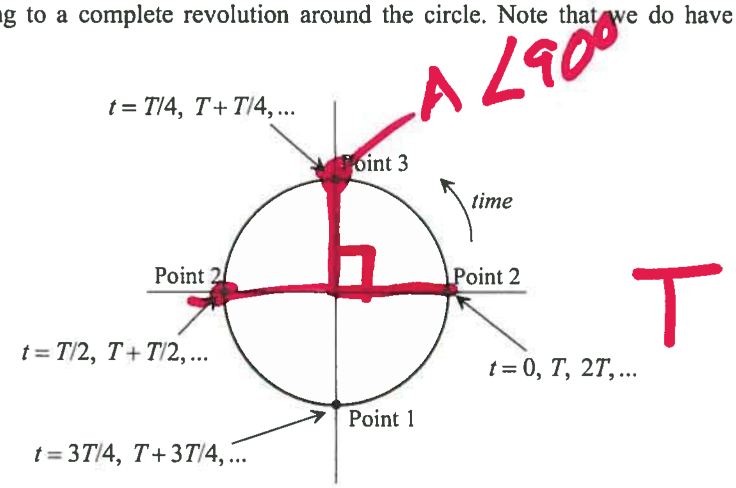


Figure 1.2 Using a circle to describe the movement of the pendulum in Fig. 1.1.

6)

limitations when representing the movement of the pendulum with this circle. For example, what is the amplitude of the sinewave (what is the relative position of the pendulum along the arched path)? We'll address these concerns in a moment. For now let's write, assuming we are using radian angular units,

$$Position = \sin\left(2\pi \cdot \frac{t}{T}\right) = \sin(2\pi f_o \cdot t) \tag{1.3}$$

This function, the sine function, tells us our relative position along the arc (the argument of this function is the angle which relates to the position on the circle in Fig. 1.2). Point 2 corresponds to the function having a value of 0 (and times, $t = 0$, $T/2$, T, $3T/2$, $2T$, ...), Point 3 to a value of +1, and Point 1 corresponds to −1. Finally, remember that the values of the sine function in Fig. 1.1.b, and Eq. (1.3), are determined empirically from measured data (e.g., plotting the pendulum's position along the arched path against time).

Describing Amplitude in the x-y Plane

Examine the sinewave in Fig. 1.3a. For the moment we won't concern ourselves with the actual distance the pendulum swings. In Fig. 1.3b we represent the sinewave, at Point i (and Point vi), as a zero length vector along the x-axis (the amplitude of the sinewave is 0 at this point in time). As we move towards Point ii in Fig. 1.3a the length of the vector

$1\angle 90° = 0 + 1 \cdot j$

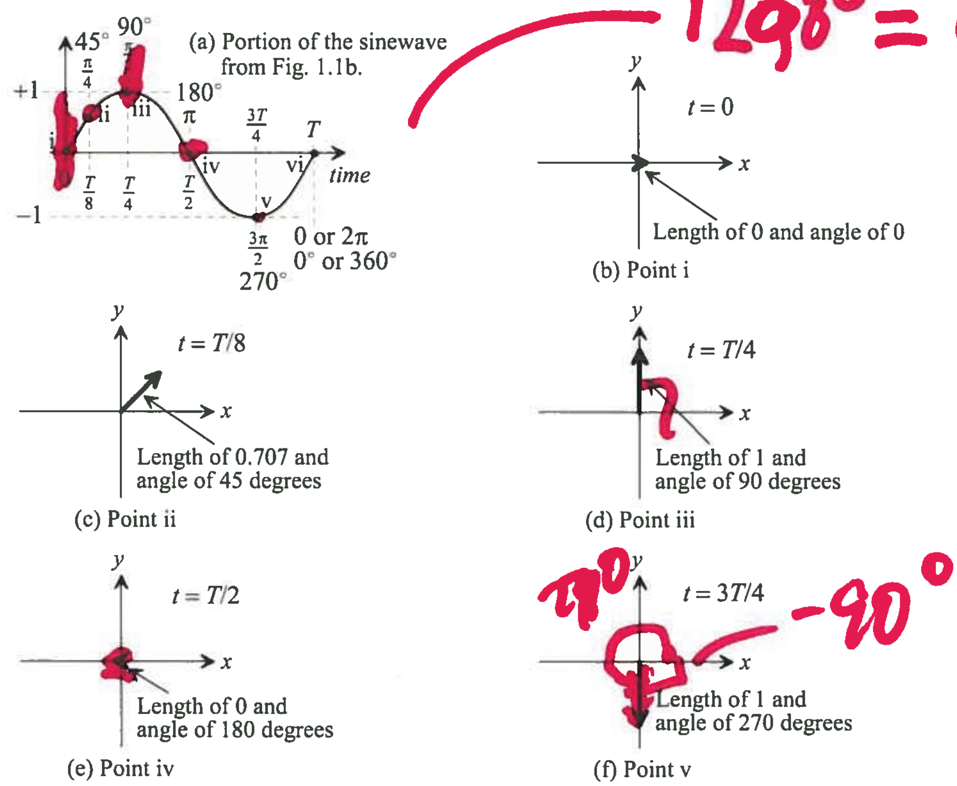


Figure 1.3 A vector swinging around the x-y plane changing both length and angle is used to represent a sinewave.

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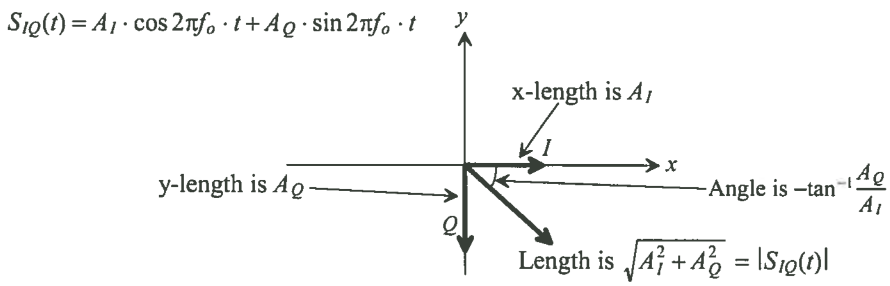


Figure 1.6 Again, showing how an I/Q signal can be represented in the x-y plane.

to simplify the math! To move towards this goal we'll develop the complex, or *z*-, plane and the frequency-domain representation of signals.

1.1.2 The Complex (z-) Plane

Let's attempt (and fail using the *x-y* plane) to simplify our mathematical description of the *IQ* signal given in Eq. (1.10). Recall the following Taylor series expansions

$$e^k = 1 + k + \frac{k^2}{2!} + \frac{k^3}{3!} + \frac{k^4}{4!} + \ldots \tag{1.12}$$

$$\cos k = 1 - \frac{k^2}{2!} + \frac{k^4}{4!} - \frac{k^6}{6!} + \frac{k^8}{8!} - \ldots \tag{1.13}$$

$$\sin k = k - \frac{k^3}{3!} + \frac{k^5}{5!} - \frac{k^7}{7!} + \frac{k^9}{9!} - \ldots \tag{1.14}$$

We can now write

$$\cos k + \sin k = 1 + k - \frac{k^2}{2!} - \frac{k^3}{3!} + \frac{k^4}{4!} + \frac{k^5}{5!} - \frac{k^6}{6!} - \frac{k^7}{7!} + \frac{k^8}{8!} + \frac{k^9}{9!} - \ldots \tag{1.15}$$

Comparing Eq. (1.15) to Eq. (1.12) we see that we are close to writing the Taylor's series for e^k. Why is this important? Perhaps the simplest explanation is that if we can represent sinewaves using exponentiation, then multiplying two sinewaves, or shifting a sinewave in time, can be performed using simple addition (of exponents).

The question now is how do we modify things to ensure that all terms are added so that Eq. (1.15) matches Eq. (1.12)? Let's look at the first discrepancy $(-1)\cdot\frac{k^2}{2!}$. The only way to change the polarity of this term is take the square root of -1 and move it inside with k^2. As the reader may know instead of writing $\sqrt{-1}$ for all of these terms we simplify things and write

$$j = \sqrt{-1} \tag{1.16}$$

Numbers using j (or i) are called *imaginary or complex numbers* (the reason for using the name imaginary will be explained in Ex. 1.1). Imaginary numbers are invaluable for time-shifting and scaling sinusoidal signals. We now rewrite Eq. (1.12) using j as

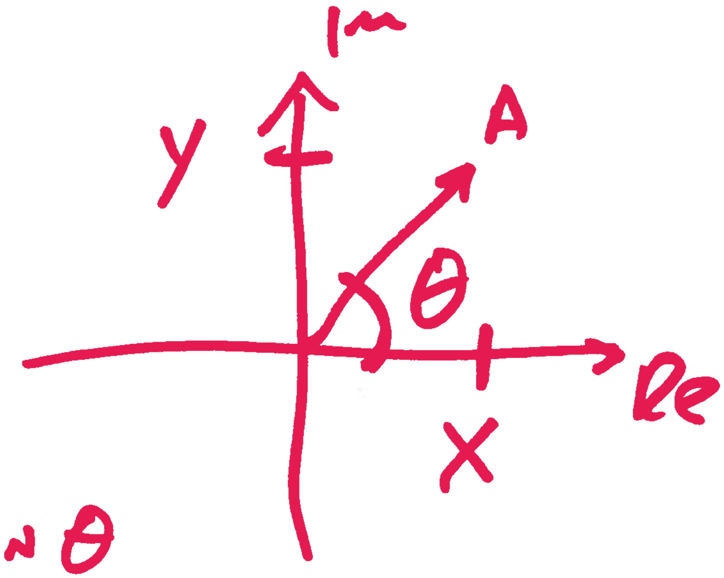
$$e^{jk} = 1 + jk - \frac{k^2}{2!} - j\frac{k^3}{3!} + \frac{k^4}{4!} + j\frac{k^5}{5!} - \frac{k^6}{6!} - j\frac{k^7}{7!} + \ldots \tag{1.17}$$

8) $Re\{e^{jk}\} = \cos k + j \sin k$
Euler's identity

$$\text{Polar} = A\angle\theta$$

$$x = A\cos\theta$$

$$y = A\sin\theta$$



$$X = A\cos\theta + Aj\sin\theta$$

$$|X| = \sqrt{A^2\cos^2\theta + A^2\sin\theta} = \underline{A} \cdot \cancel{\sqrt{\cos^2\theta + \sin^2\theta}}^{\,1}$$

$$\angle X = \tan^{-1}\frac{A\sin\theta}{A\cos\theta} = \underline{\theta}$$

$$\frac{A_1\angle\theta_1}{A_2\angle\theta_2} = \frac{A_1}{A_2}\angle\theta_1 - \theta_2$$

$$A_1\angle\theta_1 + A_2\angle\theta_2 = ?$$

$$A_1\angle\theta_1 \cdot A_2\angle\theta_2$$

$$A_1A_2\angle\theta_1+\theta_2$$

$$A_1\cos\theta_1 + jA_1\sin\theta_1$$

$$+\ A_2\cos\theta_2 + jA_2\sin\theta_2$$

$$\underbrace{A_1\cos\theta_1 + A_2\cos\theta_2} + j\cdot(A_1\sin\theta_1 + A_2\sin\theta_2)$$

Impedance of C & L

$v = i \cdot R$

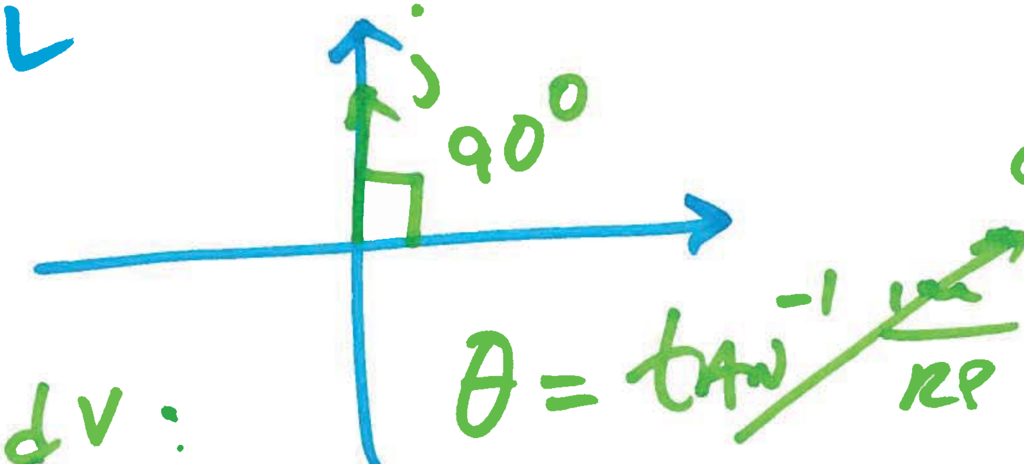
$j = \sqrt{-1}$

$$Z_C = \frac{1}{j \cdot 2\pi f \cdot C} = \frac{j}{j} \cdot \frac{1}{j\omega C} = j \cdot \frac{-1}{\omega C}$$

$v = i \cdot \frac{1}{j\omega C}$

$$Z_L = j \cdot 2\pi f \cdot L = \underline{j\omega L + 0}$$

$v = i \cdot j\omega L$

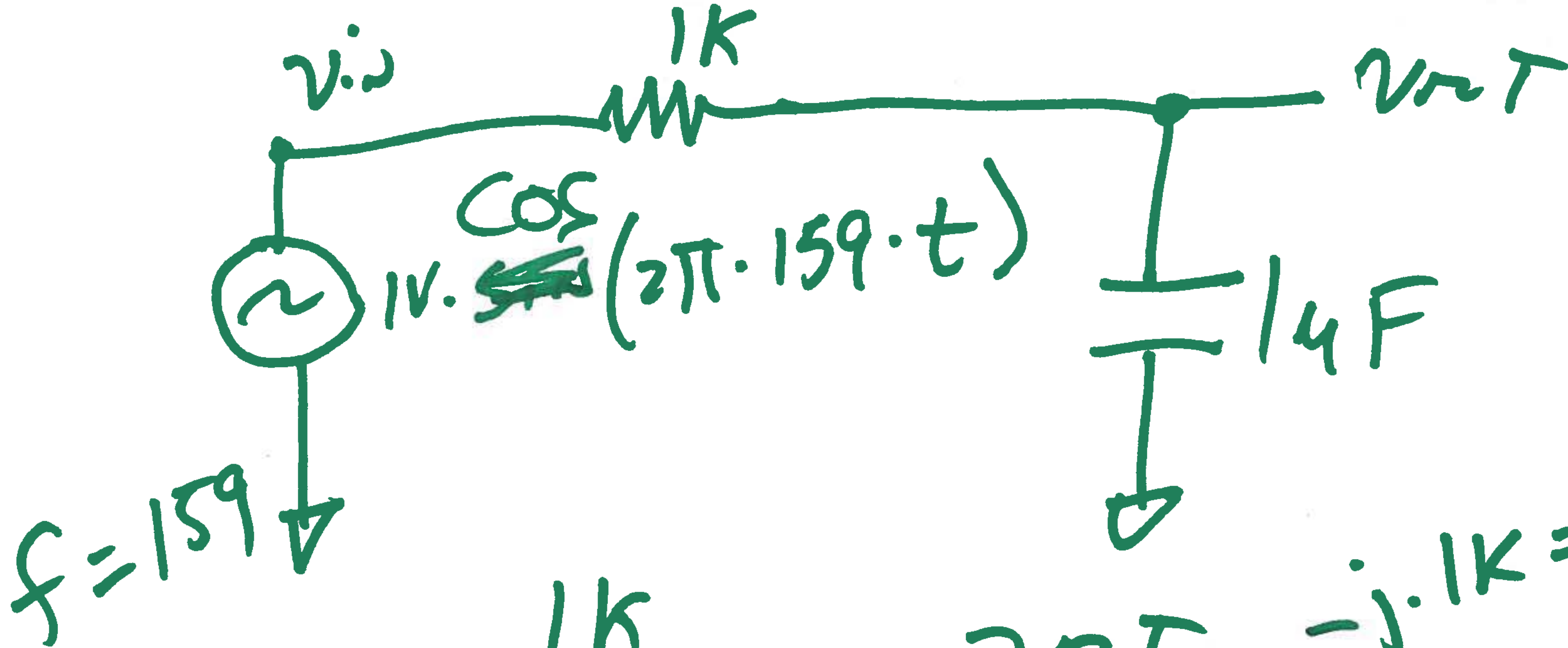


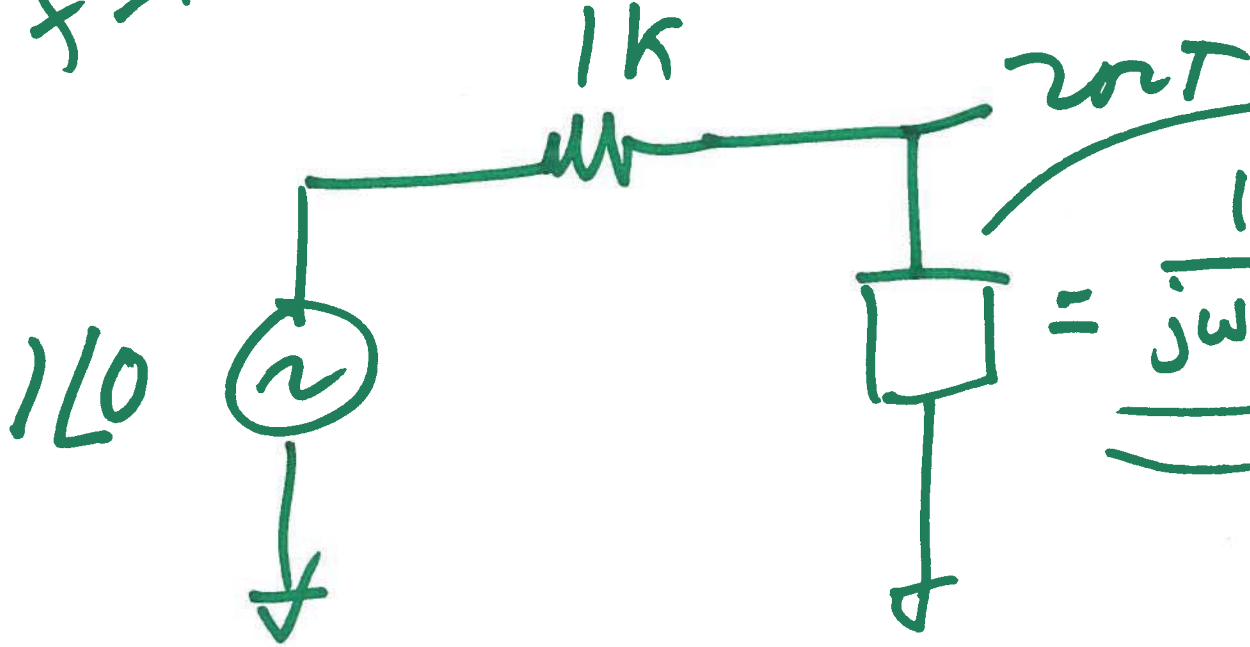
$$\theta = \tan^{-1} \frac{\omega L}{0}$$

RP

$$i = C \frac{dv}{dt}$$

$$v = L \cdot \frac{di}{dt}$$

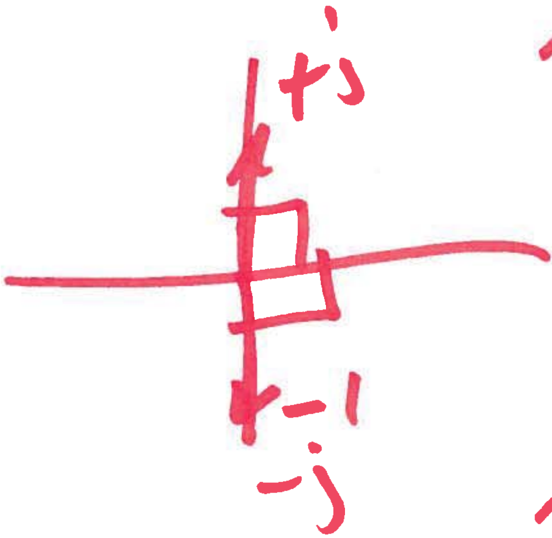




$$-j \cdot 1K = \frac{1K}{j}$$

$$= \frac{1}{j\omega C} = \frac{1}{j\,2\pi \cdot 159 \cdot 10^{-6}}$$

$$= \frac{1}{j\,10^{-3}} = \frac{1K}{j}$$



$$v_{outT} = \frac{-j\cdot 1k}{-j\cdot 1k + 1k} \cdot \cancel{v_{in}} \; 1\angle 0$$

$$v_{outT} = \frac{0 + j(-1)}{1 + j(-1)} \cdot 1\angle 0^\circ$$

-90°

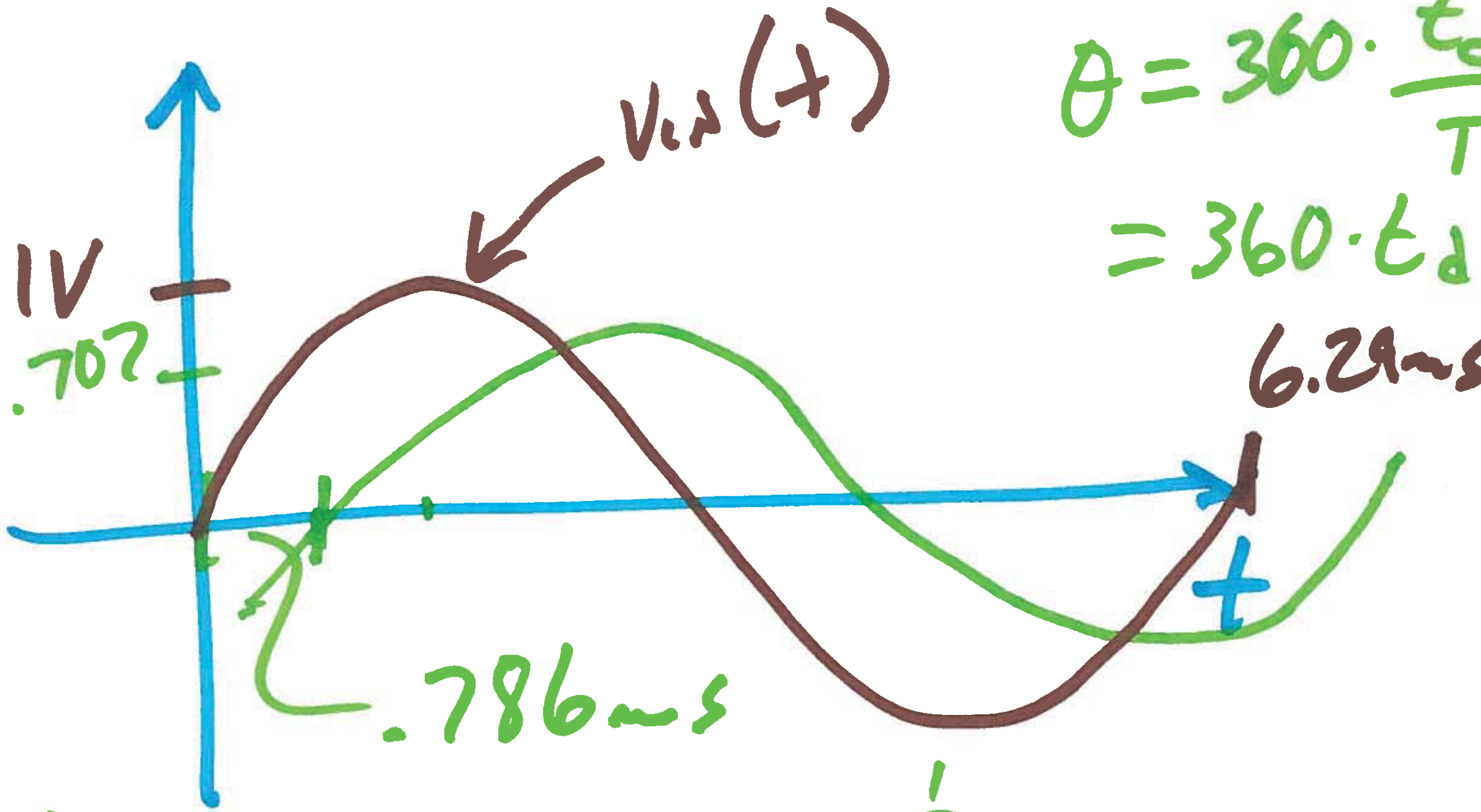
$$= \frac{\sqrt{0^2+(-1)^2}\;\angle \cancel{\tan^{-1}}\frac{-1}{0}}{\sqrt{1^2+(-1)^2}\;\angle \tan^{-1}\frac{-1}{1} \Rightarrow -45^\circ} \cdot 1\angle 0^\circ$$

$$= \frac{1\angle -90}{\sqrt{2}\angle -45} \cdot 1\angle 0^\circ = \frac{1}{\sqrt{2}}\angle -45^\circ$$

$.707\angle -45^\circ$

$$v_{outT} = 0.707 \sin\left(2\pi \cdot 159 \cdot t - \frac{\pi}{8}\right)$$

$$-45^\circ \Rightarrow \frac{\pi}{8}$$



$$\theta = 360 \cdot \frac{t_d}{T}$$

$$= 360 \cdot t_d \cdot f$$

$-45°$

$$t_d = \cancel{\frac{45°}{360°}}^{\,\frac{1}{8}} \cdot 6.29\,ms$$

$$= .786\,ms$$