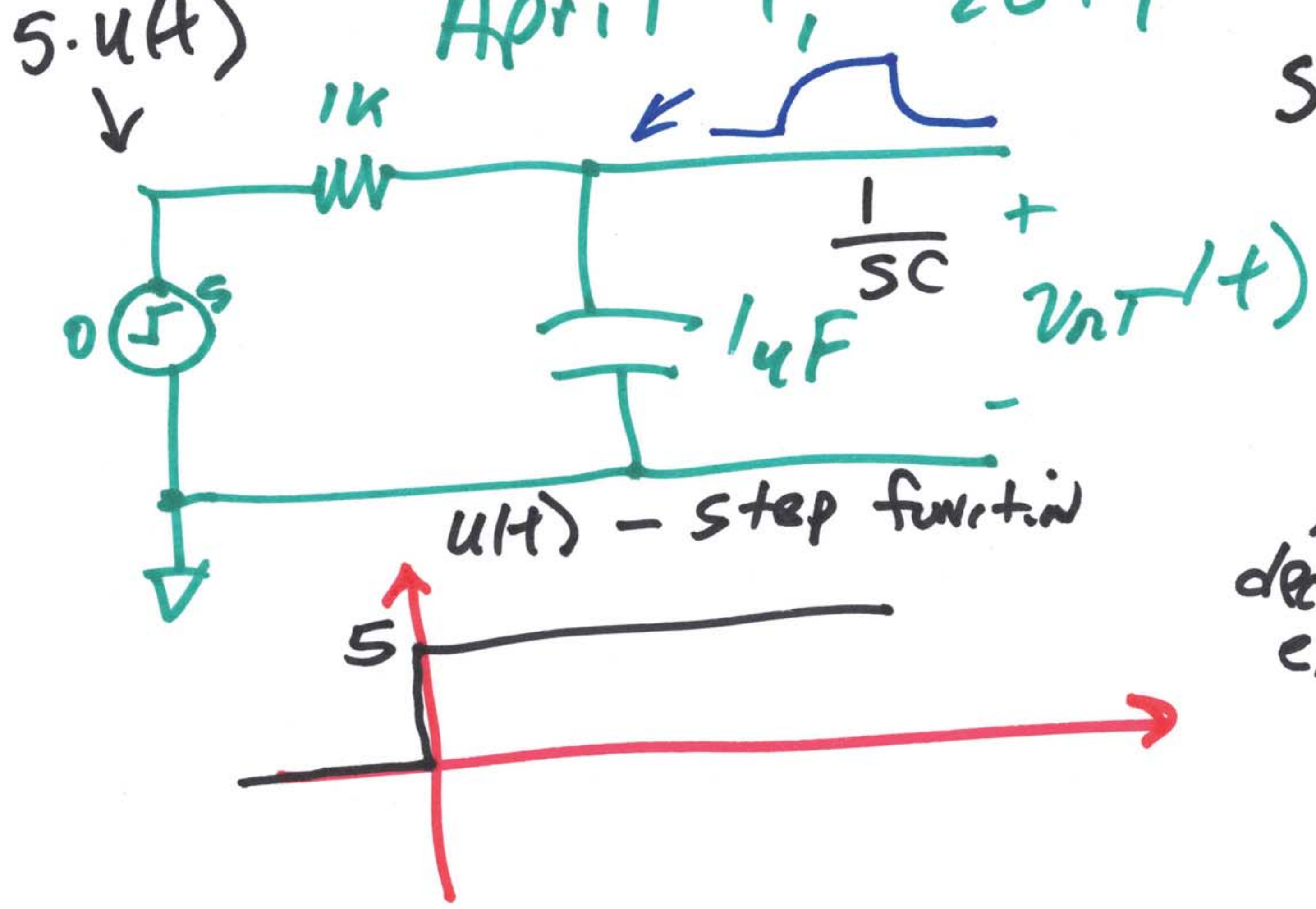
EE 221 Circuits II
Lecture 16
April 1, 2019

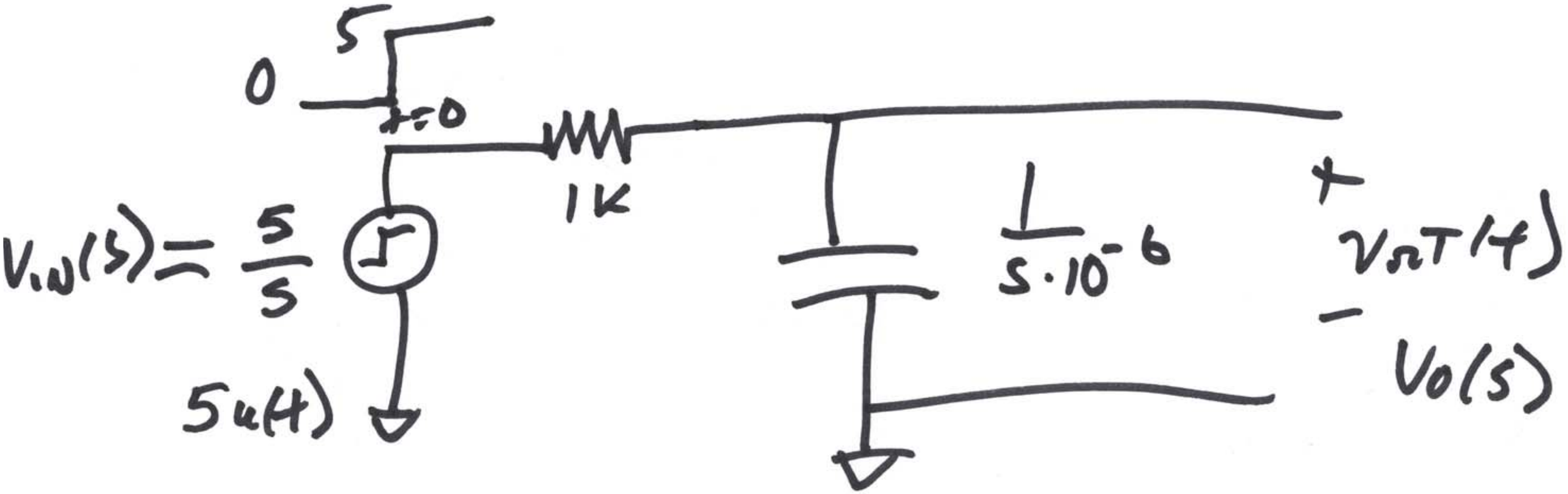


$$S = \sigma + j\underbrace{\omega}$$

$$e^{st}$$

$$e^{\sigma t}\, e^{j\omega t}$$

dec. exp. ↗ sin ↗



$$V_o(s) = \frac{\frac{1}{s\cdot10^{-6}}}{\frac{1}{s10^{-6}} + 10^3} \cdot \frac{5}{s}$$

$$= \frac{1}{1 + + s\cdot10^{-3}} \cdot \frac{5}{s}$$

$$V_o(s) = \frac{1}{1 + s\cdot 10^{-3}} \cdot \frac{5}{s}$$

$$= \frac{10^3}{s + 10^3} \cdot \frac{5}{s}$$

partial fraction expansion

$$\frac{10^3}{s+10^3}\cdot\frac{5}{\cancel{s}}\cdot\cancel{s}\Bigg|_{s=0} = \cancelto{0}{\frac{A}{s+10^3}\cdot s}\Bigg|_{s=0} + \cancelto{B}{\frac{B}{\cancel{s}}}\cdot\cancel{s} = V_o(s)$$

$$\boxed{5 = B}$$

$$\cancel{s+10^3}\cdot\frac{10^3}{\cancel{s+10^3}}\Bigg|_{s=-10^3}\cdot\frac{5}{s} = \cancel{s+10^3}\cdot\left(\frac{A}{\cancel{s+10^3}} + \cancelto{0}{\frac{B}{s}}\right)\cdot s+10^3$$

$$\boxed{A = -5}$$

3)

$$\cancel{s+10^3}\left(\frac{10^3}{\cancel{s+10^3}}\cdot\frac{5}{s}\right)\Bigg|_{s=-10^3} = \frac{A\cdot\cancel{(s+10^3)}}{\cancel{s+10^3}}\Bigg|_{s=-10^3} + \underbrace{\frac{B\cdot\cancel{s+10^3}}{s}}_{0}\Bigg|_{s=-10^3}$$

$$\cancel{10^3}\cdot\frac{5}{-\cancel{10^3}} = A$$

$$\boxed{A=-5}$$

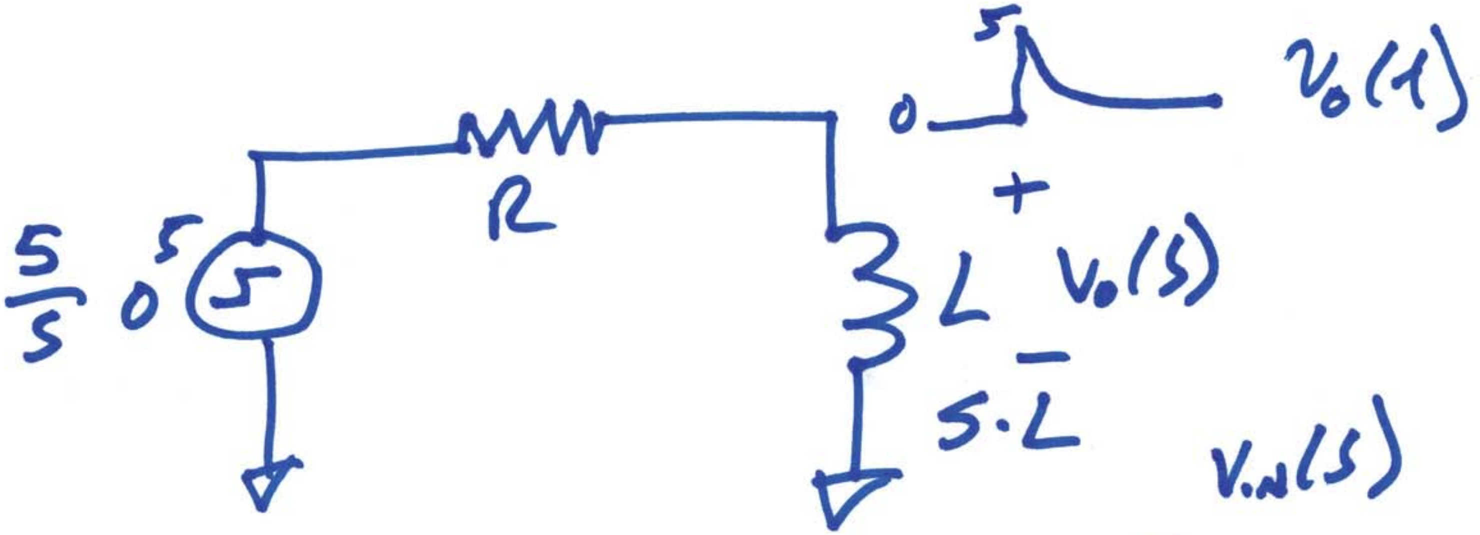
$$V_o(s) = \frac{-5}{s+10^3} + \frac{5}{s}$$

$$v_o(t) = -5e^{-10^3\cdot t}\cdot u(t) + 5u(t)$$

$$= 5\left(1-e^{-t/10^{-3}}\right)u(t)$$

$$= 5\left(1-e^{-t/10^{-3}}\right) \quad t \geq 0$$

↖



$$V_o(s) = \frac{s\cdot \cancel{L} \xrightarrow{} 1}{s\cdot \cancel{L} + \frac{R}{L}} \cdot \frac{5}{s} \qquad s=\sigma+j\omega$$

$$\frac{\cancel{s}}{\cancel{s}+\frac{R}{L}}\cdot\frac{5}{\cancel{s}}\cancel{s} = \frac{A\cdot \cancel{s}}{s+\frac{R}{L}} \xrightarrow{} 0 + \frac{B}{\cancel{s}}\cdot\cancel{s}$$

$$\boxed{B=0}$$

$$\frac{5}{\cancel{s+\frac{R}{L}}}\cdot\frac{5}{\cancel{s}} = \frac{A}{\cancel{s+\frac{R}{L}}} + \cancel{\frac{B}{s}} \xrightarrow{} 0$$

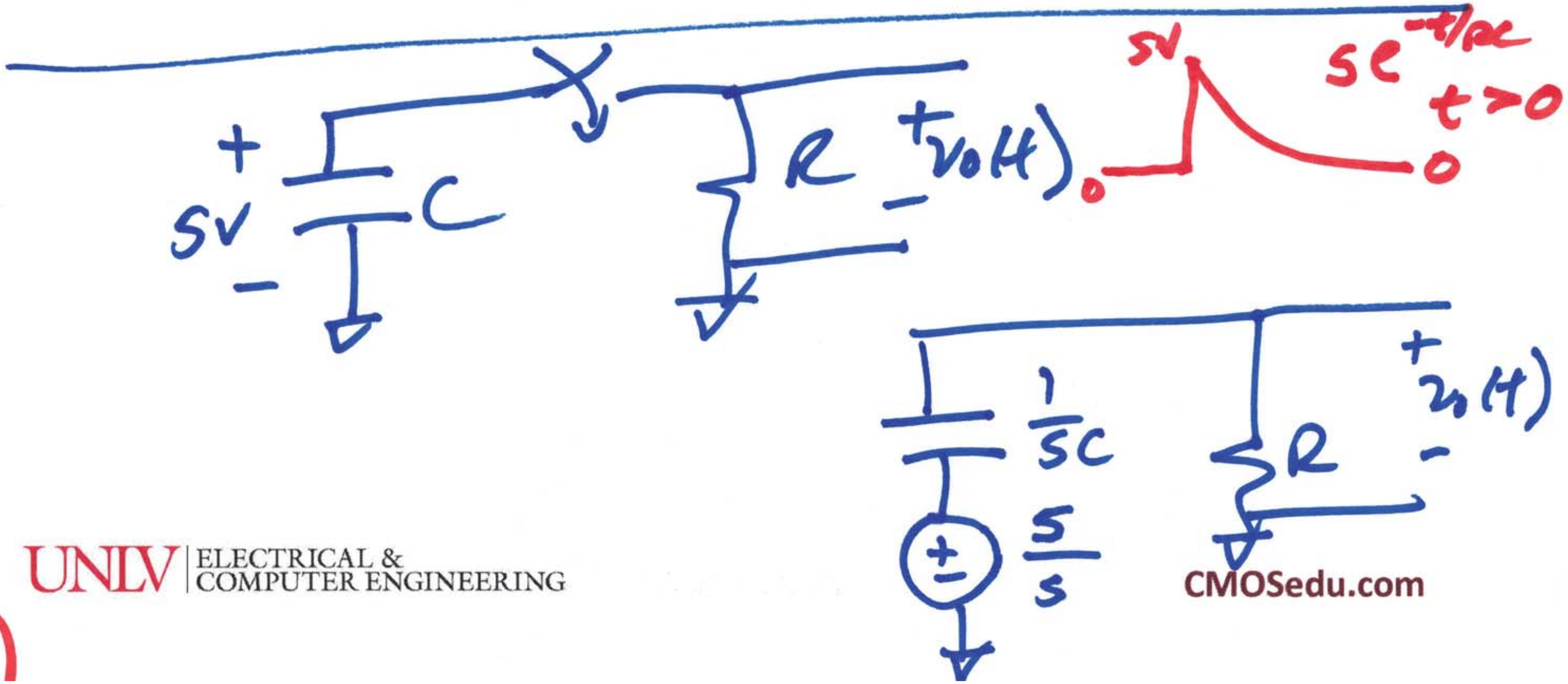
0

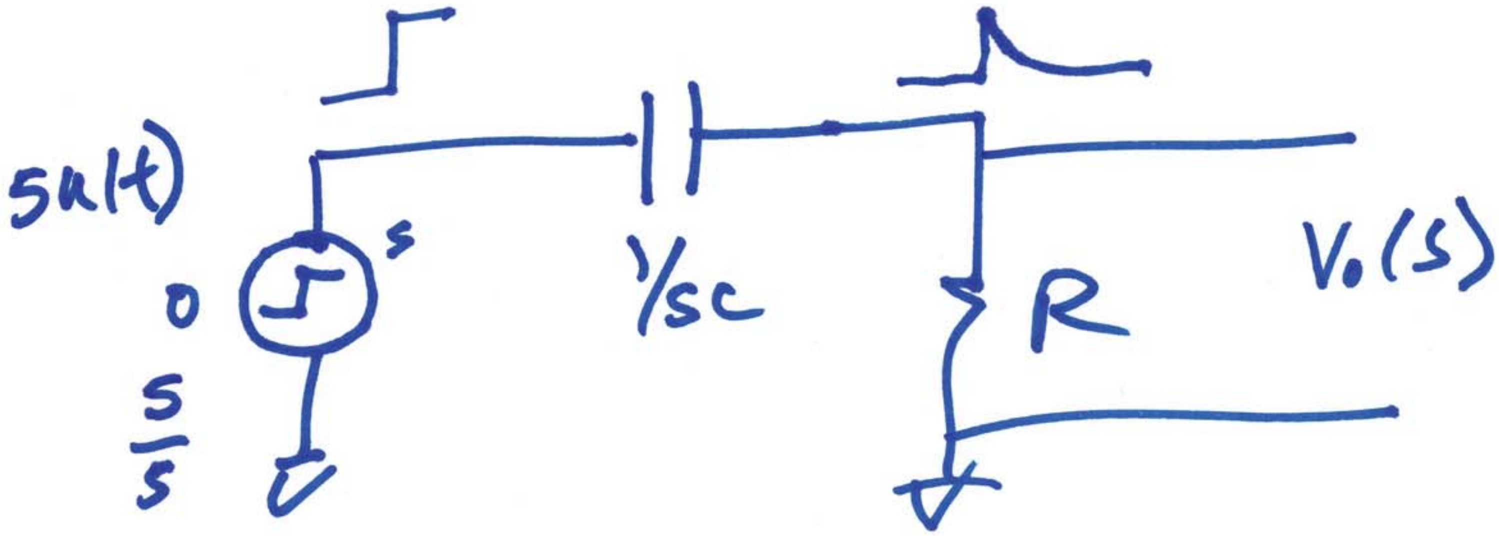
$A=5$

$$V(s) = \frac{5}{s + \frac{R}{L}}$$

$$v_0(t) = 5e^{-\frac{R}{L}\cdot t} \cdot u(t)$$

$$= 5e^{-t/L/R}, \ t > 0$$





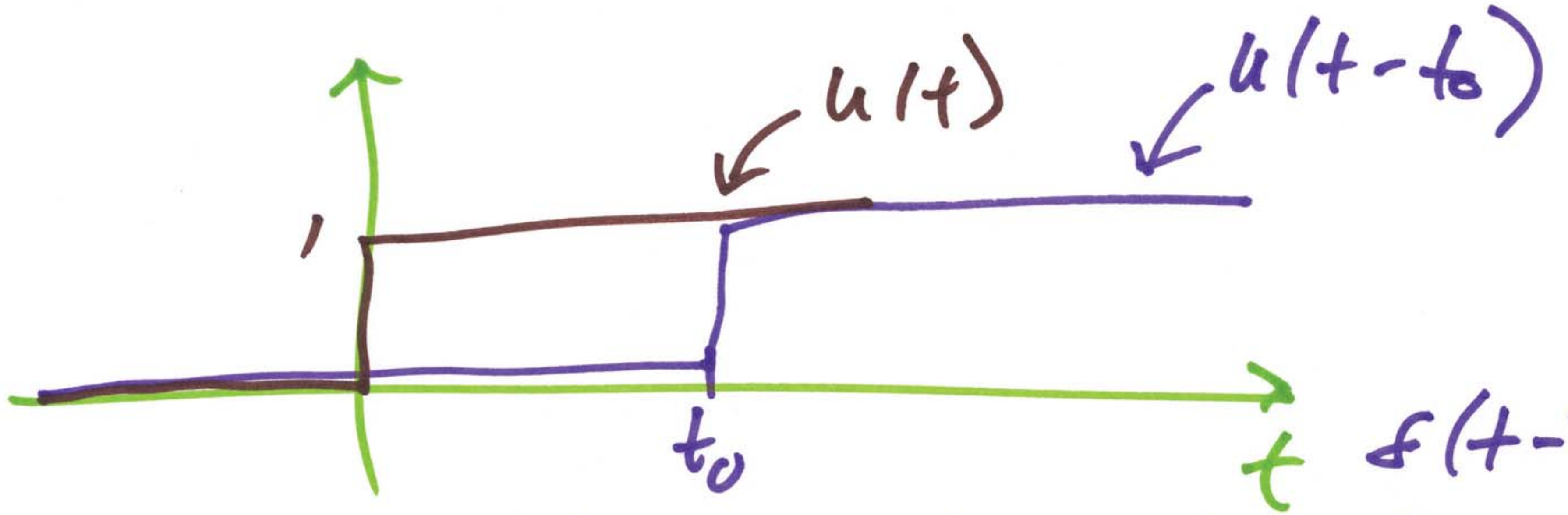
$$V_o(s) = \frac{R}{R + \frac{1}{sC}} \cdot \frac{5}{s} = \frac{\cancel{s}RC}{sRC+1} \cdot \frac{5}{\cancel{s}}$$

$$= \frac{5}{s + \frac{1}{RC}}$$

$$v_o(t) = 5e^{-\frac{1}{RC}\cdot t} \cdot u(t)$$

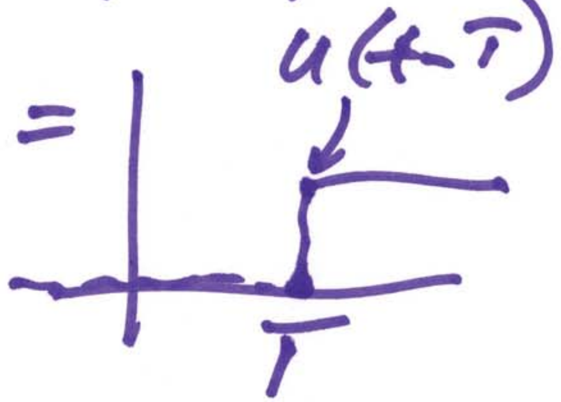
$$= 5e^{-\frac{t}{RC}} \quad t > 0$$

Unit-step function



$$\frac{du(t)}{dt} = \delta(t)$$

$$u(t-T) = \int_{-\infty}^{t} \delta(t-T) \cdot dt =$$

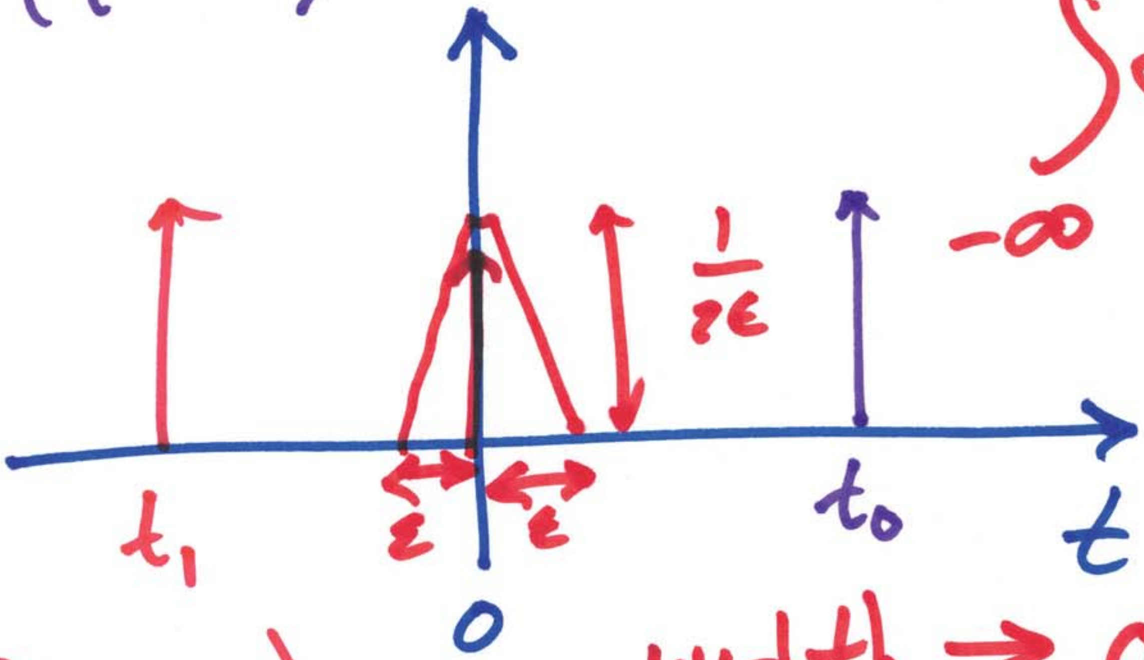


Unit impulse

$\delta(t)$

$\delta(t-t_0)$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$



$\lim_{\epsilon \to 0} = \delta(t)$

width → 0

height → ∞

$\delta(t+t_1)$

$$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$$

Definition of Laplace Transform

$$F(s) = \mathcal{L}[f(t)] = \int_{0^-}^{\infty} f(t)\cdot e^{-st}\cdot dt$$

$$s = \sigma + j\omega$$

$$\mathcal{L}^{-1}[F(s)] = f(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}\cdot ds$$

$$\mathcal{L}[u(t-t_0)]$$

$$= \int_{0^-}^{\infty} u(t-t_0)e^{-st}\cdot dt$$

$$= \int_{t_0}^{\infty} e^{-st}\cdot dt$$

$$\mathcal{L}\{u(t-t_o\} = \int_{t_o}^{\infty} e^{-st} \cdot dt$$

let $u = -st$

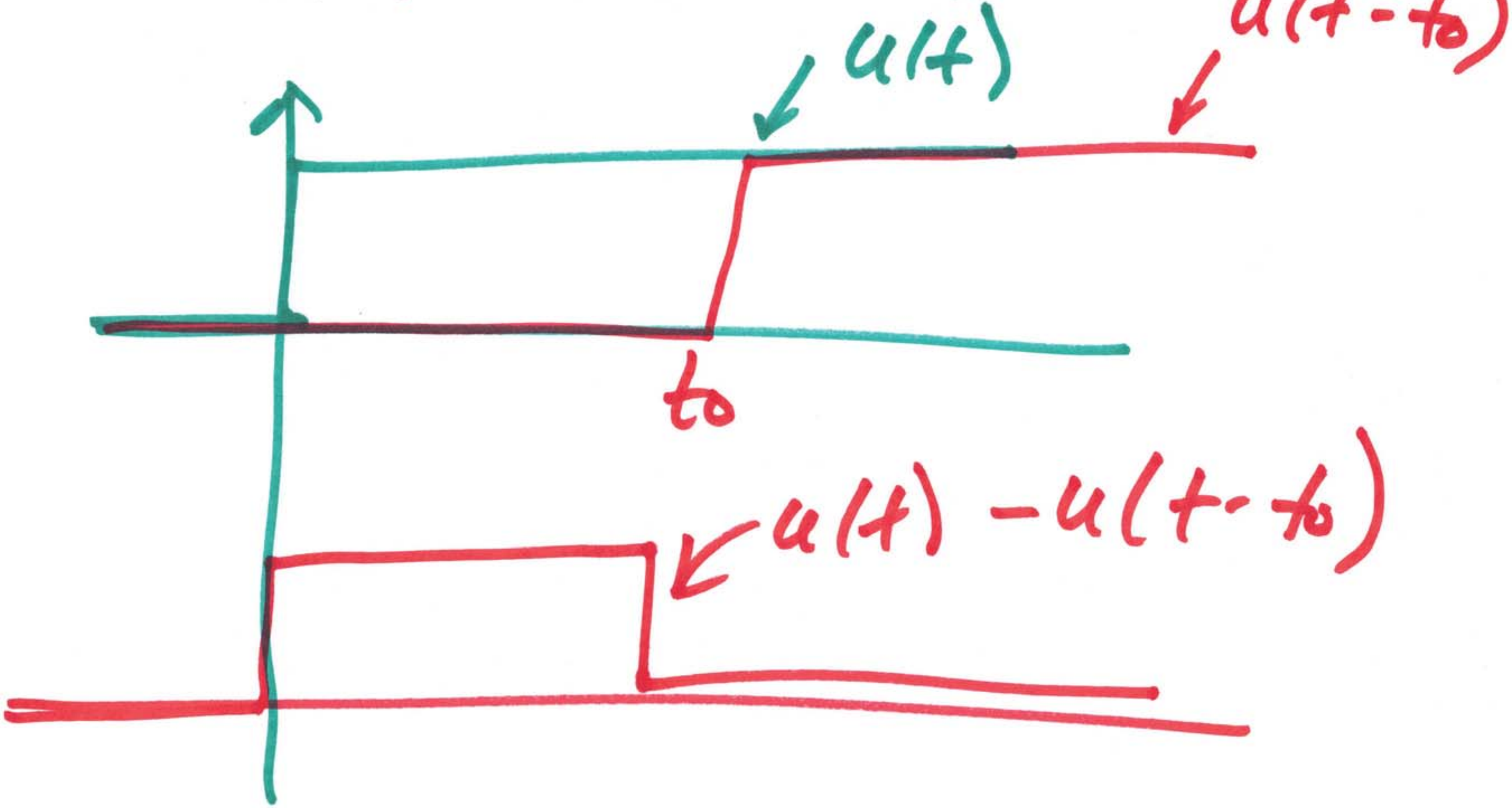
$$\frac{du}{dt} = -s \,, \quad dt = \frac{du}{-s}$$

$$= \int_{t_o}^{\infty} e^{u} \cdot \frac{du}{-s} = \frac{-1}{s} e^{u} \Big|_{-t_o s}^{-\infty}$$

$$= \frac{-1}{s}\left(e^{-\infty} - e^{-st_o}\right)$$

$$= \frac{e^{-st_o}}{s}$$

$u(t) - u(t - t_0)$



$t \cdot u(t) = r(t)$