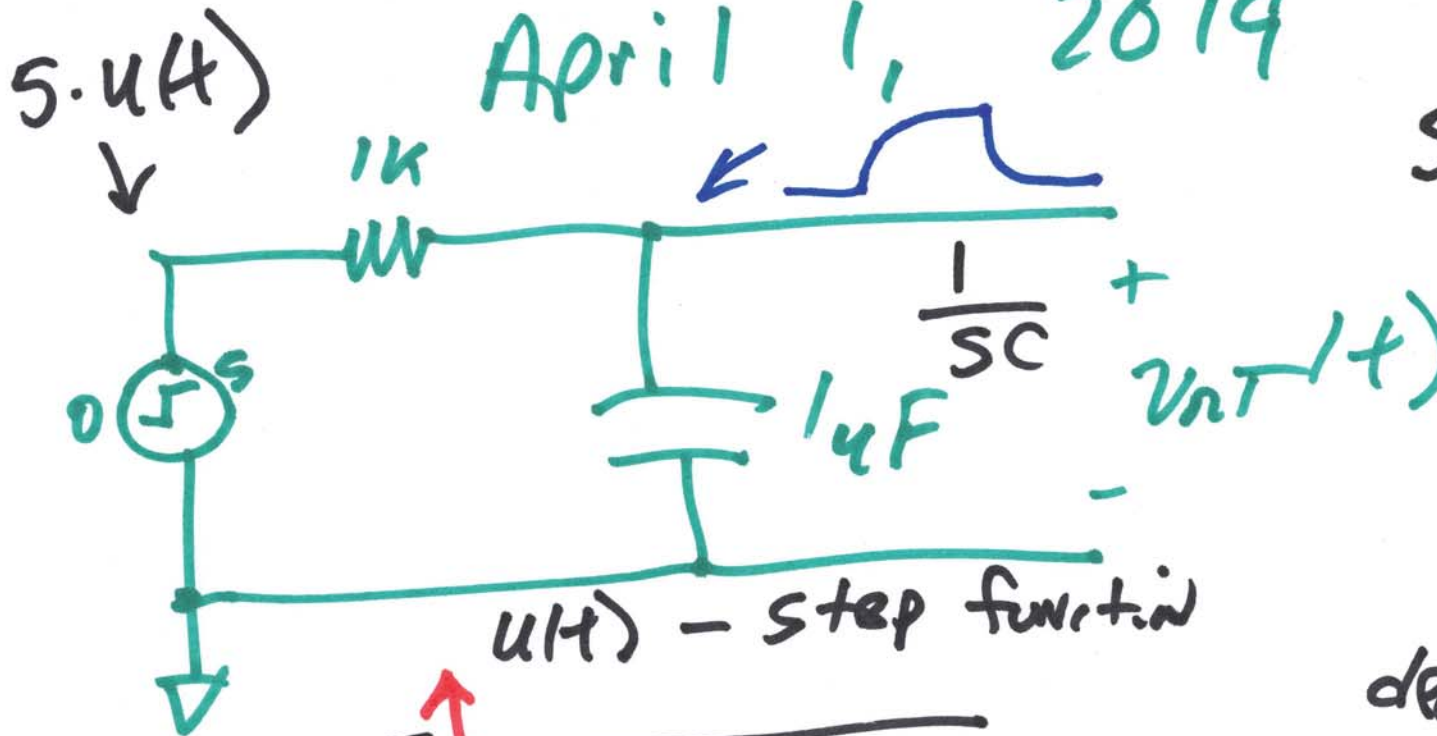


EE 221 Circuits II

Lecture 16

April 1, 2019



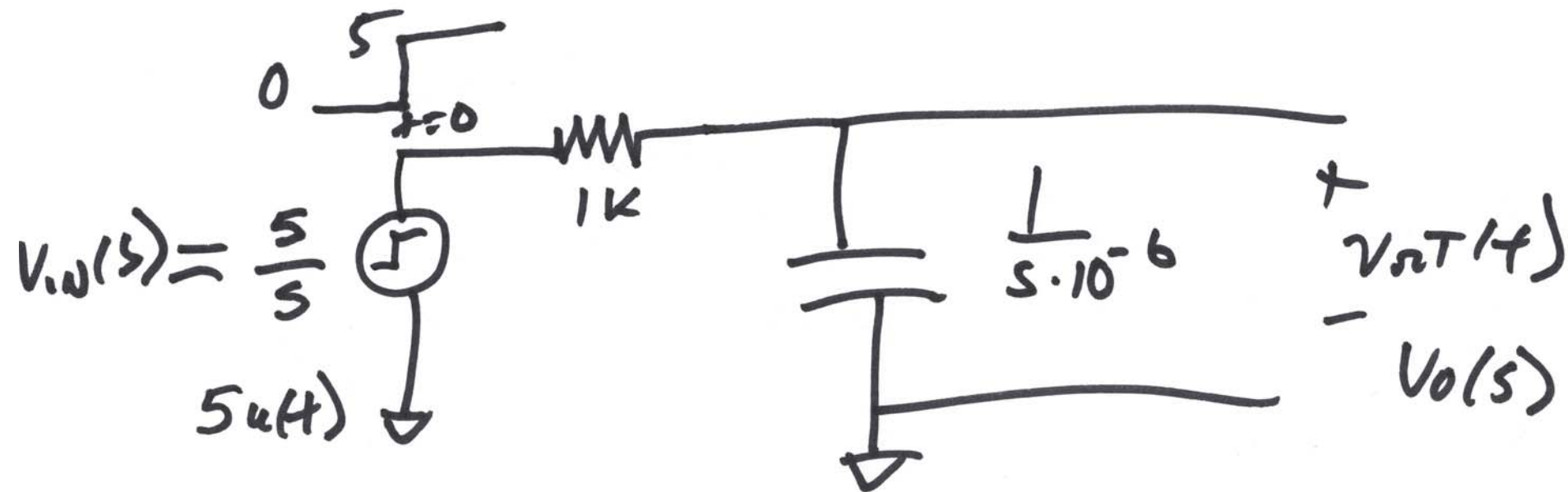
$s = \sigma + j\omega$

$e^{\sigma t}$

$e^{\sigma t} e^{j\omega t}$

dec. exp.

sin



$$\begin{aligned}
 V_o(s) &= \frac{\frac{1}{5 \cdot 10^{-6}}}{\frac{1}{5 \cdot 10^{-6}} + 10^3} \cdot \frac{5}{s} \\
 &= \frac{1}{1 + s \cdot 10^3} \cdot \frac{5}{s}
 \end{aligned}$$

7)

$$V_o(s) = \frac{1}{1 + s \cdot 10^{-3}} \cdot \frac{5}{s}$$

$$= \frac{10^3}{s + 10^3} \cdot \frac{5}{s}$$

Partial fraction expansion

$$\frac{10^3}{s + 10^3} \cdot \frac{5}{s} = \frac{A}{s} + \frac{B}{s + 10^3} = V_o(s)$$

$$s = B$$

$$\frac{10^3}{s + 10^3} \cdot \frac{5}{s} = \left(\frac{A}{s + 10^3} + \frac{B}{s} \right)$$

$$A = -5$$

3)

$$\cancel{s+10^3} \left(\frac{10^3}{\cancel{s+10^3}} \cdot \frac{s}{s} \right) = \frac{A \cdot \cancel{(s+10^3)}}{\cancel{s+10^3}} + \frac{B \cdot \cancel{s+10^3}}{s}$$


$s = -10^3$ $s = -10^3$

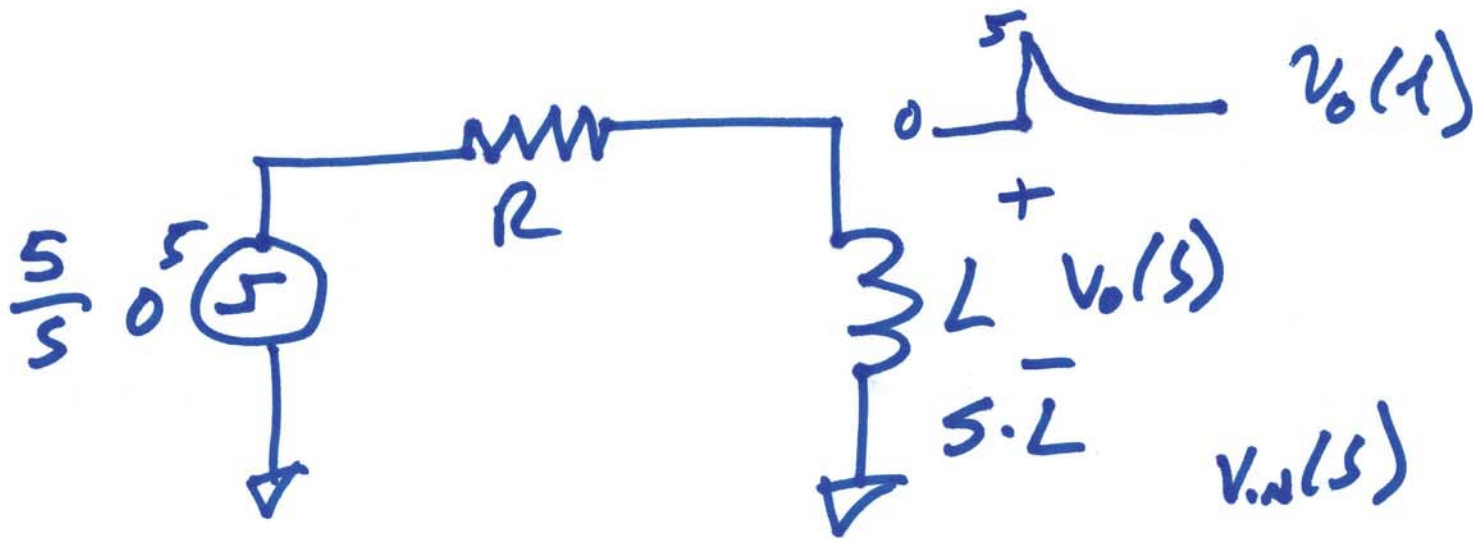
$$\cancel{10^3} \cdot \frac{s}{\cancel{-10^3}} = A$$

$A = -5$

4)

$$V_o(s) = \frac{-5}{s+10^3} + \frac{5}{s}$$

$$\begin{aligned}v(t) &= -5e^{-10^3 \cdot t} \cdot u(t) + 5u(t) \\&= 5(1 - e^{-t/10^{-3}})u(t) \\&= 5(1 - e^{-t/10^{-3}}) \quad t \geq 0\end{aligned}$$




$$v_o(s) = \frac{s \cdot L \cancel{\cdot L}}{s \cdot L + R} \cdot \frac{5}{s} \quad s = \sigma + j\omega$$

$$\frac{\cancel{s}}{\cancel{s} + \frac{R}{L}} \cdot \frac{5 \cancel{s}}{\cancel{s}} = \frac{A \cdot \cancel{s}}{\cancel{s} + \frac{R}{L}} + \frac{B \cdot \cancel{s}}{\cancel{s}}$$

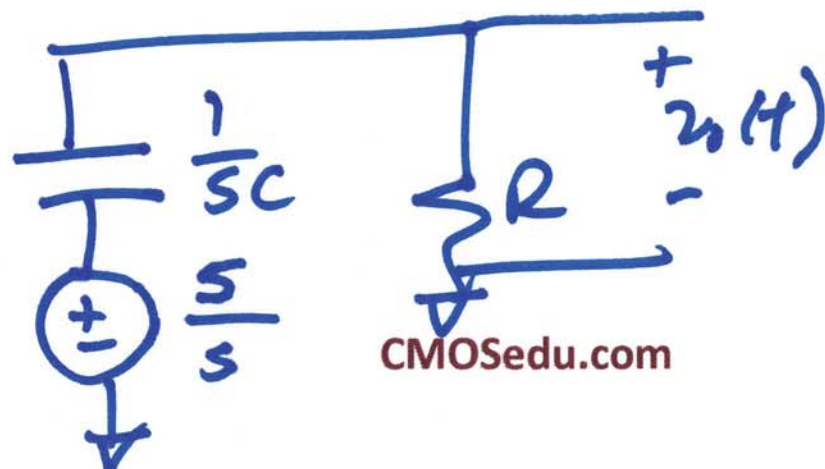
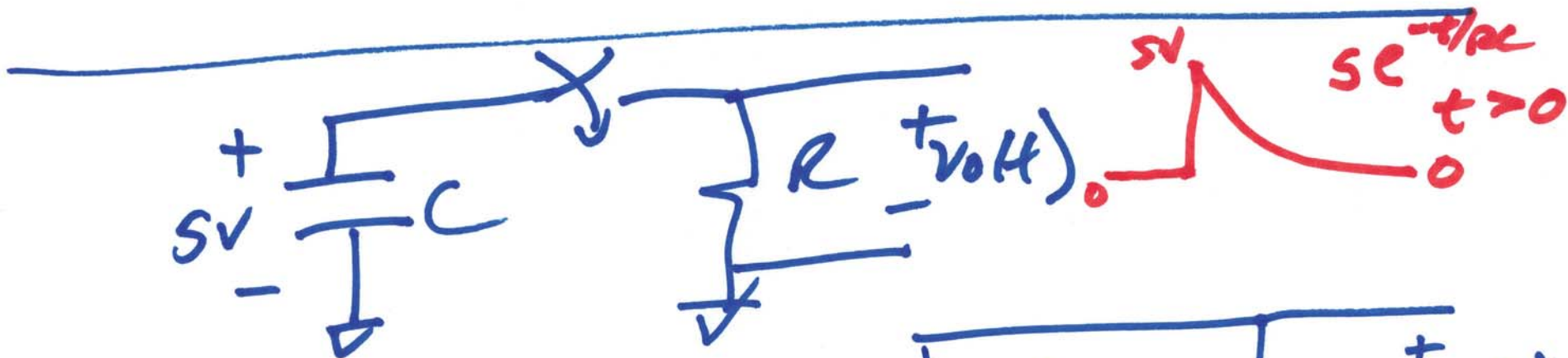
$$\frac{5}{\cancel{s} + \frac{R}{L}} \cdot \frac{5 \cancel{s}}{\cancel{s}} = \frac{A}{\cancel{s} + \frac{R}{L}} + \frac{\boxed{B=0}}{\cancel{s}}$$

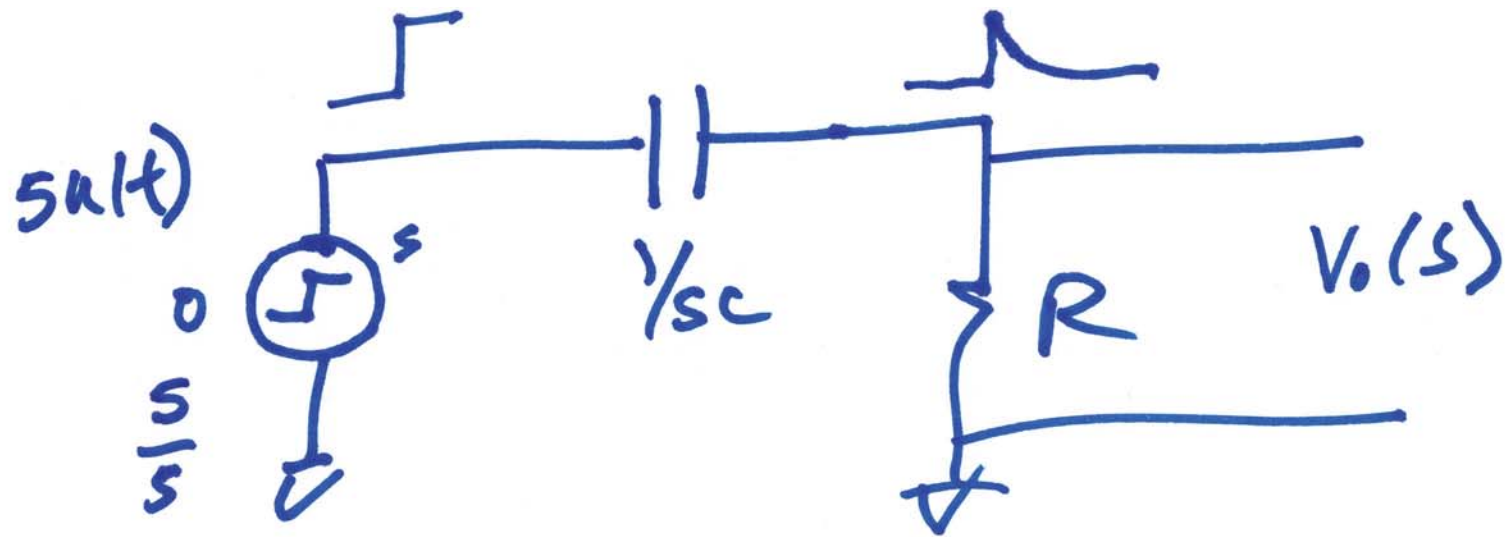
$$A = 5$$

$$V(s) = \frac{5}{s + \frac{R}{L}}$$

$$v_o(t) = 5e^{-\frac{R}{L} \cdot t} \cdot u(t)$$

$$= 5e^{-t/4R}, t > 0$$





$$V_o(s) = \frac{R}{R + \frac{1}{sc}} \cdot \frac{5}{s} = \frac{5RC}{sRC + 1} \text{ V/s}$$

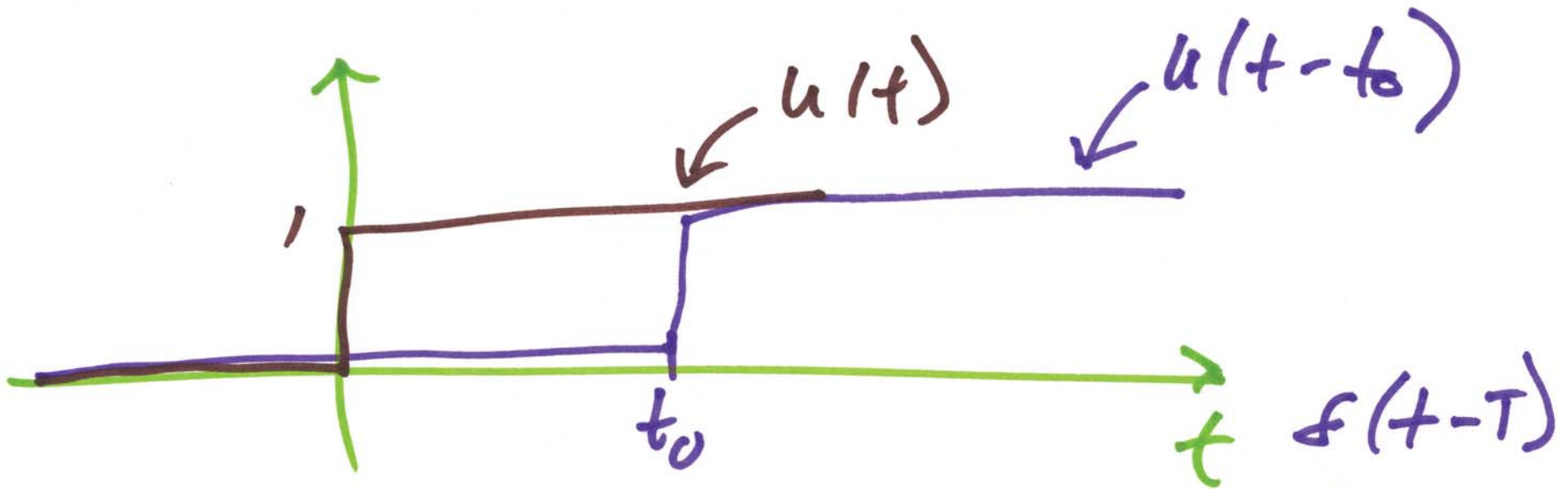
$$= \frac{5}{s + \frac{1}{RC}}$$

$$V_o(t) = 5e^{-\frac{1}{RC}t} \cdot u(t)$$

$$= 5e^{-\frac{t}{RC}} \quad t > 0$$

8)

Unit - step function



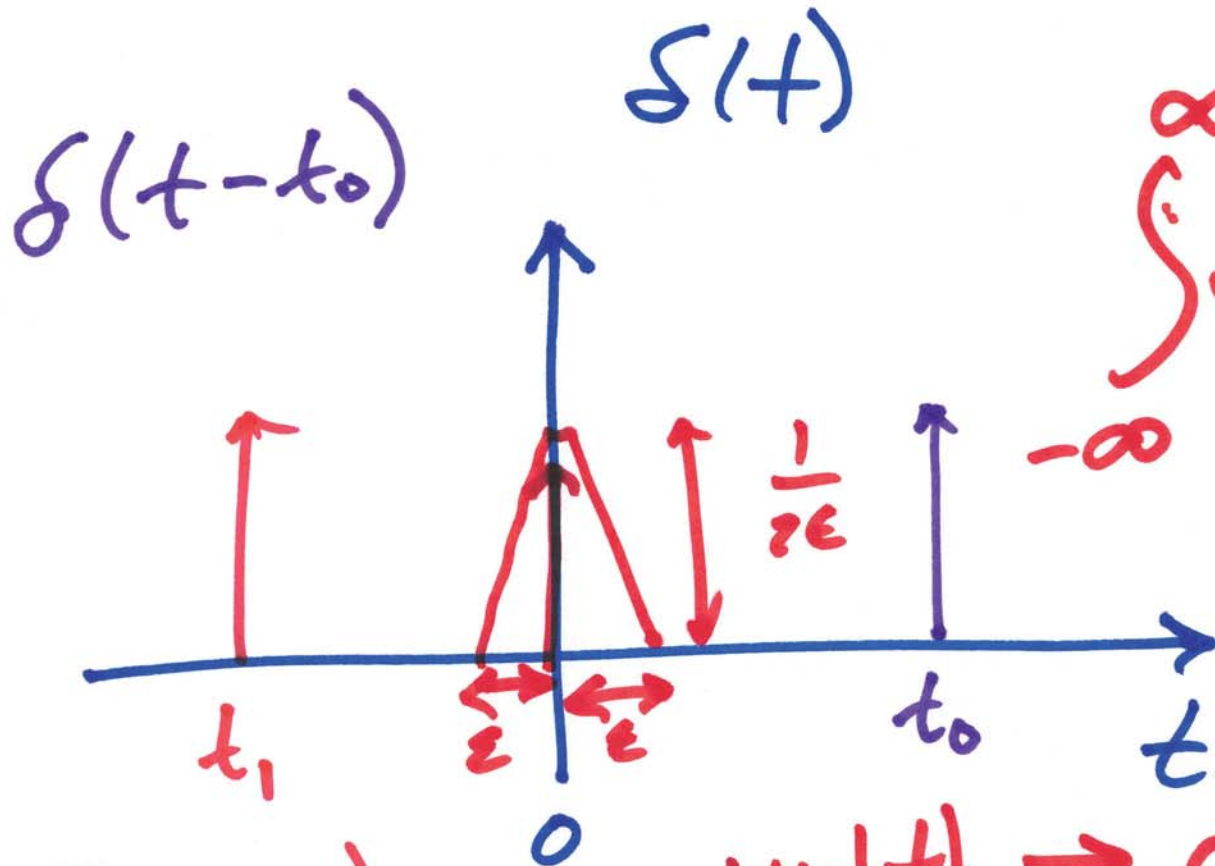
$$\frac{du(t)}{dt} = \delta(t)$$



$$u(t-T) = \int_{-\infty}^t \delta(t-T) \cdot dt =$$

A graph showing the time-shifted unit step function $u(t-T)$. The horizontal axis is labeled t and the vertical axis is labeled $f(t-T)$. The function is zero for $t < T$ and one for $t > T$. A tick mark T is shown on the horizontal axis.

Unit impulse



$\delta(t-t_0)$

$\delta(t)$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

$$\lim_{\epsilon \rightarrow 0} \delta(t) = \delta(t)$$

$\delta(t+t_1)$

width $\rightarrow 0$
height $\rightarrow \infty$

$$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$$

a)

Definition of Laplace Transform

$$F(s) = \mathcal{L}[f(t)] = \int_{0^-}^{\infty} f(t) \cdot e^{-st} \cdot dt$$

$$s = \sigma + j\omega$$

$$\mathcal{L}^{-1}[F(s)] = f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s) e^{st} \cdot ds$$

$$\mathcal{L}^* [u(t-t_0)]$$

$$= \int_{0^-}^{\infty} u(t-t_0) e^{-st} \cdot dt$$
$$= \int_{t_0}^{\infty} e^{-st} \cdot dt$$

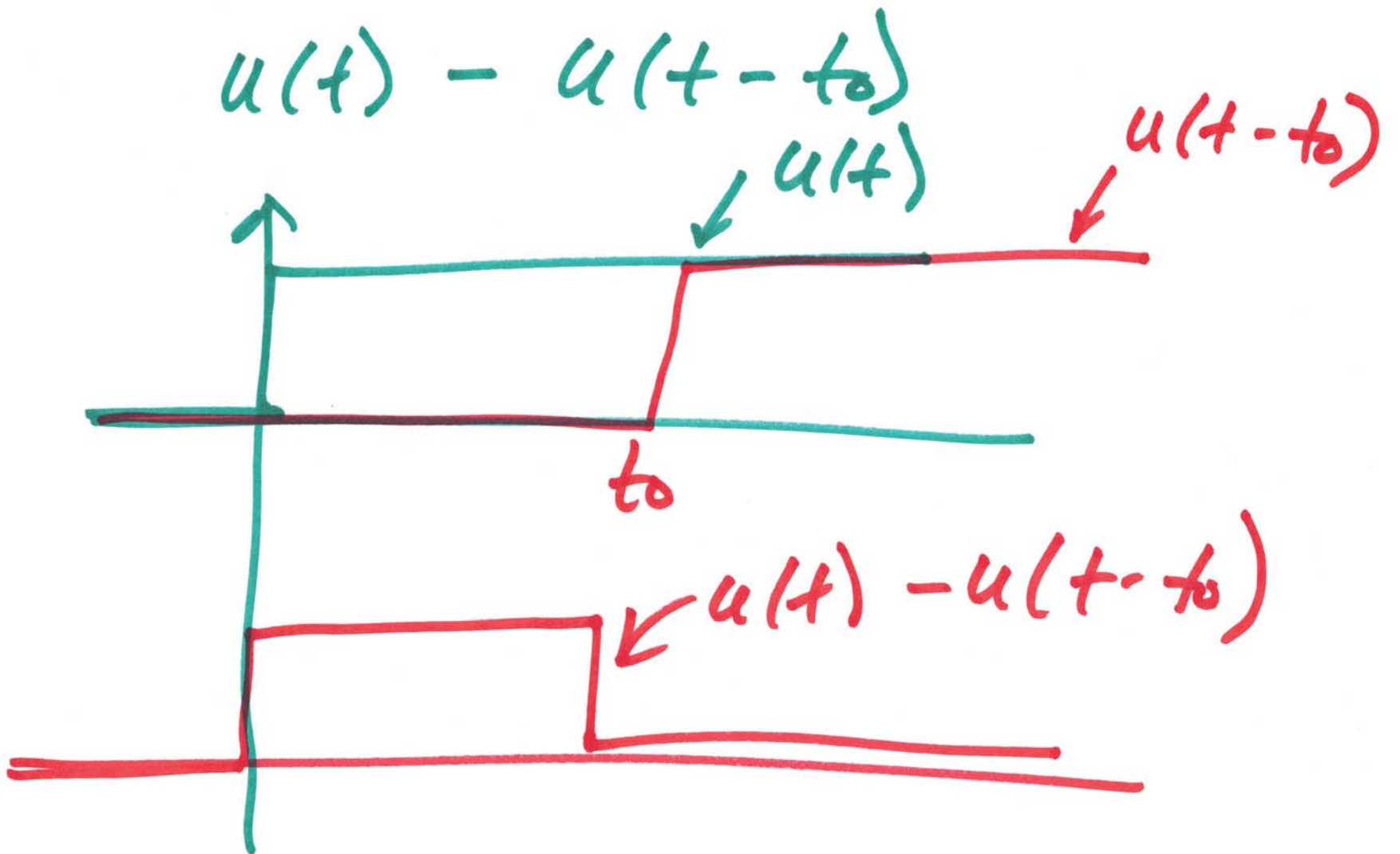
$$\mathcal{L}\{u(t-t_0)\} = \int_{t_0}^{\infty} e^{-st} \cdot dt$$

$$\text{let } u = -st$$

$$\frac{du}{dt} = -s, \quad dt = \frac{du}{-s}$$

$$= \int_{t_0}^{\infty} e^u \cdot \frac{du}{-s} = -\frac{1}{s} e^u \Big|_{-t_0 s}^{\infty}$$

$$= -\frac{1}{s} (e^{-\infty} - e^{-st_0})$$
$$= \frac{e^{-st_0}}{s}$$



$$t \cdot u(t) = r(t)$$

